Algebra 2 Lesson 3-1:  
Solving Linear Systems by Graphing 

Essential Question: 
How do I solve linear systems by graphing?
Linear system of equations - a linear system is two or more equations together.

Solution to a system - an ordered pair that satisfies both equations in the system. It is also the point where the graphs of the two equations intersect.

Ex: Graph the linear system and estimate the solution. Then check the solution algebraically.

\[ \begin{align*}
4x + y &= 8 & \text{Equation 1} \\
2x - 3y &= 18 & \text{Equation 2}
\end{align*} \]

Step 1: Solve the equations for y.

\[
\begin{align*}
4x + y &= 8 \\
-4x &\quad -4x \\
y &= -4x + 8
\end{align*}
\]

\[
\begin{align*}
2x - 3y &= 18 \\
-2x &\quad -2x \\
-3y &= -2x + 18 \\
-3 &\quad -3 \\
y &= \frac{2}{3}x - 6
\end{align*}
\]

Step 2: Graph the equations using slope-intercept form.

Step 3: The point where the lines intersect is the point (3, -4), which is the solution.

Check:

\[
\begin{align*}
\text{Equation 1} & \quad 4x + y = 8 \\
4(3) + (-4) &\quad \frac{2}{3} \quad 8 \\
12 - 4 &\quad \frac{2}{3} \quad 8 \\
8 &\quad \checkmark
\end{align*}
\]

\[
\begin{align*}
\text{Equation 2} & \quad 2x - 3y = 18 \\
2(3) - 3(-4) &\quad \frac{2}{3} \quad 18 \\
6 + 12 &\quad \frac{2}{3} \quad 18 \\
18 &\quad \checkmark
\end{align*}
\]
consistent- a system that has at least one solution.

inconsistent- a system that has no solution.

independent- a consistent system that has exactly one solution.

dependent- a consistent system that has infinitely many solutions.

**Number of Solutions of a Linear System**

The relationship between the graph of a linear system and the system's number of solutions is described below.

- **Exactly one solution**: Lines intersect at one point; consistent and independent
- **Infinitely many solutions**: Lines coincide; consistent and dependent
- **No solution**: Lines are parallel; inconsistent
systems with infinitely many solutions

Solve the system. Then classify the system as consistent and independent, consistent and dependent, or inconsistent.

\[ \begin{align*}
4x - 3y &= 8 & \text{Equation 1} \\
8x - 6y &= 16 & \text{Equation 2}
\end{align*} \]

Step 1: Solve both equations for y.

\[ \begin{align*}
4x - 3y &= 8 \\
-4x &= -4x \\
-3y &= -4x + 8 \\
\text{divide each side by -3} & \quad \text{divide each side by -6}
\end{align*} \]

\[ \begin{align*}
-3y &= -4x + 8 \\
y &= \frac{4}{3}x - \frac{8}{3}
\end{align*} \]

\[ \begin{align*}
8x - 6y &= 16 \\
-8x &= -8x \\
-6y &= -8x + 16 \\
\text{divide each side by -6} & \quad \text{divide each side by -6}
\end{align*} \]

\[ \begin{align*}
-6y &= -8x + 16 \\
y &= \frac{4}{3}x - \frac{8}{3}
\end{align*} \]

Step 2: Since the equations are the same, the graphs would be the exact same line, so there are infinitely many solutions.

The system would be classified as consistent (because it has solutions) and dependent (because it has infinitely many solutions).
systems with no solution-

Solve the system. Then classify the system as consistent and independent, consistent and dependent, or inconsistent.

\[
\begin{align*}
2x + y &= 4 & \text{Equation 1} \\
2x + y &= 1 & \text{Equation 2}
\end{align*}
\]

Step 1: Solve both equations for \( y \).
\[
\begin{align*}
2x + y &= 4 \\
-2x &= -2x \\
y &= -2x + 4
\end{align*}
\]
\[
\begin{align*}
2x + y &= 1 \\
-2x &= -2x \\
y &= -2x + 1
\end{align*}
\]

Step 2: The graphs of the lines would be parallel because the lines have the same slope. There would be no solution because the lines never intersect.

The system would be classified as inconsistent because there is no solution.

Summary: To solve a linear system by graphing, first make sure that all of the equations are in slope-intercept form. Then graph the lines and find the point where they intersect (if any).
Using a graphing calculator to solve linear systems

**Question:** How can you solve a system of linear equations using a table?

\[
\begin{align*}
y &= 2x + 4 \\
y &= -3x + 44
\end{align*}
\]

**Equation 1**  
**Equation 2**

**Step 1: Enter equations**  
Press \( Y= \) to enter the equations. Enter Equation 1 as \( y_1 \) and Equation 2 as \( y_2 \).

**Step 2: Make a table**  
Set the starting \( x \)-value of the table to 0 and the step value to 1. Then use the table feature to make a table.

**Step 3: Find the solution**  
Scroll through the table until you find an \( x \)-value for which \( y_1 \) and \( y_2 \) are equal. The table shows \( y_1 = y_2 = 20 \) when \( x = 8 \).

- The solution of the system is (8, 20).